Layered partitions of planar graphs

In the beginning there was...

Mi. Pilipczuk & Siebertz '18 Every planar graph G has a vertex partition $\mathcal{P}$ into geodesics such that $G/\mathcal{P}$ has treewidth $\leqslant 8$

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Let G be a connected planar graph and let T be a rooted BFS tree of G. Then G has a vertex partition $\mathcal{P}$ into *vertical paths* of T such that $G/\mathcal{P}$ has treewidth $\leqslant 8$.

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Corollary Every planar graph is a subgraph of $H \boxtimes P$ for some graph H with treewidth $\leqslant 8$ and some path P

Applications

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Planar graphs have bounded queue-number

Dujmović, Esperet, J., Walczak, Wood '19 Planar graphs have bounded nonrepetitive chromatic number

Proof: Partitioning planar graphs

Key lemma. Suppose

- G^+ plane triangulation
- T rooted spanning tree of G^+ with root on outer-face
- cycle C partitioned into vertical paths $P_1, \ldots, P_k$, with $k \leqslant 6$
- G near-triangulation consisting of C and everything inside.

Then G has a partition $\mathcal{P}$ into vertical paths where $P_1, \ldots, P_k \in \mathcal{P}$ s.t. $= G/\mathcal{P}$ has a tree-decomposition in which every bag has size at most 9 and some bag contains all vertices corresponding to $P_1, \ldots, P_k$.

Proof: Bounded queue-number

Dujmovic, Morin, Wood '05 If H has treewidth k then $\mathrm{qn}(H) \leqslant f(k)$

Wiechert '17 If H has treewidth k then $\mathrm{qn}(H) \leqslant 2^k - 1$

Proof: Bounded queue-number

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Lemma. $\mathrm{qn}(H \boxtimes P) \leqslant 3\,\mathrm{qn}(H) + 1$ for every path P

Proof: Bounded nonrepetitive chromatic number

Kundgen & Pelsmayer '08 If H has treewidth k then $\chi_{NR}(H) \leqslant 4^k$

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Key definition: *Strongly* nonrepetitive chromatic number χ_{SNR}

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Key definition: *Strongly* nonrepetitive chromatic number χ_{SNR}

Dujmović, Esperet, J., Walczak, Wood '19 If H has treewidth k then $\chi_{SNR}(H) \leqslant 4^k$

Lemma. $\chi_{SNR}(H \boxtimes P) \leqslant 4 \cdot \chi_{SNR}(H)$ for every path P

Variant

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Every planar graph is a subgraph of $H \boxtimes P \boxtimes K_3$ for some planar graph H with treewidth $\leqslant 3$ and some path P

Variant

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Useful for improving bounds:

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Planar graphs have queue-number $\leqslant 49$

Dujmović, Esperet, J., Walczak, Wood '19 Planar graphs have nonrepetitive chromatic number $\leqslant 768$

Felsner, Micek, Schroeder '19+ Planar graphs have p-centered colorings with $O(p^3 \log(p))$ colors

Generalizations

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Every graph of Euler genus g is a subgraph of $H \boxtimes P \boxtimes K_{\max\{2g,3\}}$ for some graph H of treewidth $\leqslant 4$ and for some path P

Generalizations

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Using the structure theorem for graphs excluding a fixed minor:

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 $\forall H\ \exists k, a$ s.t. every H-minor free graph G can be obtained by clique-sums of graphs $G_1, \ldots, G_t$ s.t. for $i \in \{1, \ldots, t\}$,

$$G_i \subseteq (H_i \boxtimes P_i) + K_a,$$

for some graph H_i with treewidth $\leqslant k$ and some path P_i

Applications

Dujmović, J., Micek, Morin, Ueckerdt, Wood '19 Graphs excluding a fixed minor have bounded queue-number

Dujmović, Esperet, J., Walczak, Wood '19 Graphs excluding a fixed minor have bounded nonrepetitive chromatic number

Applications

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Using the structure theorem for graphs excluding a fixed *topological* minor:

Dujmović, Esperet, J., Walczak, Wood '19 Graphs excluding a fixed topological minor have bounded nonrepetitive chromatic number

Open problems

Class $\mathcal{G}$ has *strongly sublinear separators* (a.k.a. *polynomial expansion*) if $\exists \varepsilon > 0$ s.t. every n-vertex graph in $\mathcal{G}$ has $O(n^{1-\varepsilon})$ balanced separators

Do graphs in such a class have bounded queue number?
Bounded nonrepetitive chromatic number?

What about a structure theorem?